EFFECT OF DUVAL’S COGNITIVE MODEL ON GEOMETRICAL REASONING AND ACHIEVEMENT IN RELATION TO GEOMETRY SELF-EFFICACY

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CHAPTER VI
SUMMARY AND CONCLUSIONS

In the preceding chapter, introduction to the problem, development of tools, method of the study and interpretation of results were discussed. The present chapter focuses around brief description of the purpose, design and procedure of the study along with conclusions and suggestions for further research.

Introduction

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge”

(NCTM, 2000)

No Nation can attain any technological breakthrough without well planned and effective implementation of a mathematics education. Mathematics is the means of sharpening the individual’s mind, shaping his reasoning ability and developing his personality, hence, its immense contribution to the general and basic education of the people of the world (Asiedu-Addo & Yidana, 2000). So, mathematics teacher should therefore provide students with stimulating and wide-range of mathematical learning experiences that will develop their reasoning ability/skills and knowledge.

Yet today, as in the past, many students struggle with mathematics and become disaffected as they continually confront obstacles to engagement. In order to break this pattern it is imperative, therefore, that we have to understand what effective mathematics teaching looks like. The quality of teaching and learning mathematics has been one of the major challenges and concerns of educators. Instructional design is an effective way to alleviate problems related to the quality of teaching and learning mathematics. It is important for educators to adopt instructional design techniques to attain higher achievement rates in mathematics (Rasmussen & Marrongelle, 2006). Considering students’ needs and comprehension of higher-order mathematical knowledge, instructional design provides a systematic process and a framework for analytically planning, developing, and adapting mathematics instruction (Saritas, 2004).

In words of Education Commission (1964-66) “In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations”. Commenting on the prevailing
situation in schools, it is observed that in the average school today instruction still confirms to a mechanical routine, continues to be dominated by the old besetting evil of verbalism and therefore remains dull and uninspiring. Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic and strategies.

Teaching models prescribe tested steps and procedures to effectively generate desired outcomes. The number of emerging models and the ones that have emerged is uncountable. Each emerging new model either explores a new approach or attempts a modification of the conventional ones as to cater the uniqueness of individuals. Most importantly, any teaching model should optimize learning experiences to the needs of each learner by carefully exploring the learning problems and offering tailored assistance.

6.1 FAMILIES OF COGNITIVE MODEL OF TEACHING

As a mathematical domain, geometry is to a large extent concerned with specific mental entities, the geometrical figures. During the past twenty five years several (In recent years), a growing number of cognitive psychologists have shown a renewed interest in design issues, and have tested out their ideas by developing prototype teaching models. These teaching models differ from most educational innovations in that they are well-grounded in cognitive learning theory. Examples include John Anderson's intelligent tutors (Anderson, 1987) and Brown and Palincsar's reciprocal teaching method for teaching reading (Rosenshine & Meister, 1994); Duval’s cognitive analysis of geometrical thinking (Duval, 1998); Fischbein’s theory of figural concepts (Fischbein, 1993); van hiele model of geometry thinking(van hiele, 1986).

Historically, three major theoretical model (theoretical perspective) for cognitive growth relevant to explain the mathematics learning processes are:

- Cognitive Development model
- Cognitive model
- Cognitive science model

These families of cognitive models have been further classified into various types as shown in figure 6.1 below:
Above mentioned models under different families of models of teaching aim at the development of different aspects of human personality i.e. social, personal, informational and behavioural. Since education is meant for all round development of child’s personality, no single model can be selected for his or her development. All of them will have to be employed according to the requirements of the situation.

6.2 DUVAL’S COGNITIVE MODEL

According to Duval (1998), any model of mathematics learning in which different ways of reasoning are organised according to a strict hierarchy is inappropriate. Rather than being representative of higher (or lower) level of thinking, he argues that different kinds of cognitive activity have their own specific and independent development. He explains that geometry involves three kinds of cognitive processes in relation to discursive processes which fulfil specific epistemological functions is:

- **Visualisation processes:** Visualisation processes with regard to space representation for illustration of a statement, for the heuristic exploration of a complex situation, for a synoptic glance over it or for a subjective verification;

- **Construction processes:** Construction processes by tools, construction of configurations can work like a model in that the actions on the representative and the observed results are related to the mathematical objects which are represented;
Reasoning processes: Reasoning in relationship to discursive processes for extension of knowledge, for proof, for explanation.

Duval pointed out that these different processes can be performed separately. For example, visualisation does not necessarily depend on construction. Even if construction a leads to visualisation, construction processes actually depends only on the connection between relevant mathematical properties and the constraints of the tools being used. Similarly, even if visualisation can be an aid to reasoning (for instance, by aiding the finding of a proof) but visualization can also be misleading. However, Duval argues, “these three kinds of cognitive processes are closely connected and their synergy is cognitively necessary for proficiency in geometry” (ibid, p38). Approaching geometry from a cognitive point of view, he has distinguished four cognitive apprehensions connected to the way a person looks at the drawing of a geometrical figure: perceptual, sequential, discursive and operative apprehension.

6.3 GEOMETRICAL REASONING

One of aspect of basic competencies mathematics is reasoning which include step of higher mathematics thinking; include capacity for thinking logically and systematically. Reasoning is a process to reach logical conclusion based on facts and relevant resources. Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts (National Council of Teachers of Mathematics, 2000).

Mathematical reasoning is the ability to think coherently and logically and draw inferences or conclusions from mathematical facts known or assumed (Mansi, 2003).

According to Duval, “the word ‘reasoning’ is used in a very broad range of meanings. Any move, any trial and error, any procedure to solve a difficulty is often considered as a form of reasoning.” (ibid). For developing students’ reasoning in geometry, Duval introduced three cognitive processes in geometry involved in proof/reasoning, namely

“A Purely Configural Process” – operative apprehension,

“A Natural Discursive Process” – in ordinary speech through description, explanation and argumentation,

“A Theoretical Discursive Process” – this is performed through deduction.
He argued that purely configural process can be embedded into a natural discourse but cannot be embedded into a theoretical discourse. The gap between the purely configural process and the theoretical discursive process in proof contributes to most of the difficulties encountered by students. Duval further argued that there is always a gap between the natural discursive process, which is closer to ‘everyday’ language and some things can be designed as ‘figural reasoning’, and the theoretical discursive process, which is performed in a purely symbolical register or in the natural language register.

Thus, reasoning in relationship to discursive processes for extension of knowledge, for proof, for explanation. Formal reasoning abilities have been identified as essential abilities for students achievement and success in advanced science and mathematics courses.

6.4 ACHIEVEMENT

The problem, why students achieve or fail to achieve in the school has always interested psychologist and educators. Achievement is one of the most important goals of education. Academic achievement is a measure of understanding or skills in a specified subject or group of subjects combined.

Best & Kahn, (2006) Achievement tests attempt to measure what an individual has learned his or her present level of performance. They are used in diagnosing strengths and weaknesses and as a basis for awarding prizes, scholarships, or degrees. Many of the achievement tests used in schools are nonstandard zed, teacher-designed tests.

Random house dictionary (2011) achievement connotes final accomplishment of something noteworthy, after much effort and often in spite of obstacles and discouragements.

Achievement as performance in school or college is done through tests, usually through teachers made tests. Teacher made test are used to identify specific objectives that have previously been taught and to evaluate the degree to which students have mastered these objective. A the continuous process of assessing achievement, school have relied on larger scale evaluation of students achievement. Thus, assessment of academic achievement helps both the students and teacher to know where they stand.
Factors Affecting Achievement

Academic achievement is a multidimensional, multifaceted phenomenon. Factors that are responsible achievement of the students has been grouped into two broad classes: subjective factors and objective factors.

Also factors affecting achievement have been classified into following categories of their sources:

Cognitive factors: like intelligence, creativity, ability, learning rate, reasoning ability etc.

Home related factors: socio-economic status, family size, birth order, gender bias, parental involvement, and parental expectation working status of parents.

Affecting factors: like values, interests, self-efficacy, perseverance, stress etc.

6.5 MATHEMATICS SELF-EFFICACY

According to Bandura (cited in Pajares, 2002) self-efficacy forms the basis for human motivation, perseverance and behaviour. Human action and perseverance is determined by failure or success in a task or action. Learners in the Maths class may react quite differently to the results that each receives in an assessment. Two learners, who have achieved similar results after putting in equal effort, might be affected in different ways. A learner who normally achieves well might be disillusioned by the lower performance, whereas a learner who does not normally achieve well, will no doubt be highly excited by this increased performance (Pajares, 2002).

Source to Improve Self-Efficacy in Maths Learners

Pajares (2002) outlines four primary sources that he considers to be major contributors to the formation of self-efficacy beliefs that is (a) mastery experience, (b) vicarious experience, (c) emotional states and (d) verbal/social persuasions.

Mastery Experience

Bandura emphasises that mastery experience is the most influential source of self-efficacy beliefs in the learner. The learner’s past successes or failures are the most reliable way of assessing the self-efficacy of a learner. He contends that exposing or engaging Maths learners in authentic mastery experiences results in
successful Maths experiences and that in turn enhances learners self efficacy in that aspect of Maths.

**Vicarious Experience**

Learner’s self-efficacy can be boosted by observing others perform (model) mathematical tasks. If the learner observes someone (who has similar attributes as the learner), succeed in an activity it contributes to creating in the learner the belief that the learner is also capable of accomplishing the task with the same degree of success (Pajares, 2002).

**Verbal/Social Persuasion**

Using social persuasion to build learners’ self-efficacy beliefs in Maths entails using realistic judgements about learner’s performance in Maths. It can increase learners’ beliefs in their capability to perform a task with the required level of success. Verbal persuasion must be realistic if it is to work and must be reinforced by authentic experience. Convincing learners that they are capable of the Maths task at hand enables them to overcome their doubt about their capability to take up the challenge successfully. Positive persuasion should seek to empower and motivate learners through developing their self-efficacy beliefs (Bandura, 1986; Pajares, 2002).

**Somatic and Emotional States**

Learners who anticipate failure in a Maths task, and experience strong emotional feelings of stress and fear, have low self-efficacy beliefs in Maths. However, learners who expect to be successful in the task have a positive emotional welfare which enhances their self-efficacy beliefs. Learners, who are convinced that they will fail at a task, most probably will because they themselves are convinced of it (Pajares, 2002).

### 6.6 GEOMETRY SELF-EFFICACY

Bandura focused on self-efficacy in a variety of domains such as chemistry self-efficacy, computer self-efficacy, career self-efficacy, sport self-efficacy, Academic self-efficacy, teacher self-efficacy, mathematics self-efficacy belief, computer usage etc. But researchers like Pajares, Betz and Hackett (1995) have focused specifically on self-efficacy as it related to math. Self-efficacy beliefs have
already been studied in relation to a lot of aspects of mathematics learning, such as: arithmetical operations, Self-Efficacy towards Mathematics Scale Statistics Self-efficacy, problem solving and problem posing self-efficacy and Geometry self-efficacy.

Geometry self-efficacy is defined as a situational assessment of an individual's confidence in her or his ability to successfully perform or accomplish a particular geometry based task or problem.

Thus, it may be concluded that, the quality teaching and learning of mathematics depends upon use of effective instructional strategies for development of cognitive ability. Child’s Cognitive ability in turn enhances his affective arena.

The present study therefore was an attempt to see our reliance on Duval’s cognitive model over conventional where students learning and geometrical reasoning were assessed in respect of geometry self-efficacy. The study therefore was stated as follows:

6.7 STATEMENT OF THE PROBLEM

EFFECT OF DUVAL’S COGNITIVE MODEL ON GEOMETRICAL REASONING AND ACHIEVEMENT IN RELATION TO GEOMETRY SELF-EFFICACY

6.8 DELIMITATIONS OF THE STUDY

Keeping time and resource constraints in view, the study was delimited to the following:

1. The study was limited to Duval’s cognitive model of instruction and conventional group learning only in area of school Euclidean geometry.
2. The study was conducted on class VIII students studying in Govt. secondary schools situated in the union territory Chandigarh.
   - Affiliated to CBSE
   - Co-educational
   - Hindi medium
3. The sample of the study was delimited to 300 students only.
4. The impact of Duval’s cognitive model of instruction was studied mainly on geometrical reasoning performance and geometry-achievement.
5. The study was limited to Duval’s three cognitive processes:
   • Visualization processes
   • Construction processes
   • Reasoning processes

As synergy of these processes are essential for proficiency in geometry.

6. Among three cognitive processes in Duval’s cognitive model, the study was limited only to reasoning processes because most important prior of others as it will guide to more complicated and abstract aspect of visualization and construction processes to develop geometrical reasoning ability.

7. The study was limited to geometry self-efficacy only.

6.9 OBJECTIVES OF THE STUDY

The study is designed to attain the following objectives

• To develop and validate instructional material based upon Duval’s cognitive model for geometry.

• To study the effectiveness of instructional strategies i.e. Duval’s cognitive model (DCM) as against conventional group learning (CGL) on Geometrical-reasoning.

• To study the impact of geometry self-efficacy on Geometrical-reasoning.

• To study the interaction effect of instructional strategies (DCM and CGL) and geometry self-efficacy and on Geometrical-reasoning.

• To study the effectiveness of instructional strategies (DCM vs. CGL) on Achievement in geometry.

• To study the impact of geometry self-efficacy on Achievement in geometry.

• To study the interaction effect of instructional strategies (DCM and CGL) and geometry self-efficacy on Achievement in geometry.

6.10 HYPOTHESES

H0.1 There is no significant difference in the EGT scores of students learning through Duval’s cognitive model (DCM) and Conventional group learning (CGL).
H02 There is no significant difference in the EGT scores of students at different levels of geometry self-efficacy (high, intermediate and low).

H03 There is no significant interaction effect between learning situations and levels of geometry self-efficacy (GSE) to yield difference in scores of EGT.

H04 There is no significance difference in the pre-criterion scores of students learning in two instructional strategies (DCM, CGL).

H05 There is no significance difference in the pre-criterion scores of students at the three levels of geometry self-efficacy (High, Intermediate, and Low).

H06 The interaction effect of the instructional strategy and levels of geometry self-efficacy did not yield significant differences in the pre-criterion scores.

**Geometrical Reasoning (Total)**

H07 There is no significant difference in gain mean of students on geometrical reasoning (total) studying through DCM and CGL.

H08 There is no significant difference in gain mean of students on geometrical reasoning (total) of students with high, intermediate and low geometry self-efficacy groups.

H08.1 Gain means of students on geometrical reasoning of high and intermediate groups are not different.

H08.2 Gain means of students on geometrical reasoning of high and low groups are not different.

H08.3 Gain means of students on geometrical reasoning of intermediate and low groups are not different.

H09 There is no significant interaction effect of instructional strategies (DCM and CGL) with geometry self-efficacy (high, intermediate and low) of students to yield different gain means on geometrical reasoning (total).

H09.1 Through DCM, high, intermediate and low geometry self-efficacy students do not differ in their geometrical reasoning gain means.

H09.2 Through CGL, high, intermediate and low geometry self-efficacy students do not differ in their geometrical reasoning gain means.

H09.3 High geometry self-efficacy students achieve equal geometrical reasoning gain means in DCM and CGL.
H09.4  Intermediate geometry self-efficacy students achieve equal geometrical reasoning gain means in DCM and CGL.

H09.5  Low geometry self-efficacy students achieve equal geometrical reasoning gain means in DCM and CGL.

**Purely Configural Process of Geometrical Reasoning**

H010  There is no significant difference in gain mean of students on purely configural process of geometrical reasoning studying through DCM and CGL.

H011  There is no significant difference in total gain mean of students on purely configural process geometrical reasoning of students with high, intermediate and low geometry self-efficacy groups.

  H011.1  Gain means of students on purely configural process of geometrical reasoning of high and intermediate groups are not different.

  H011.2  Gain means of students on purely configural process of geometrical reasoning of intermediate and low groups are not different.

  H011.3  Gain means of students on purely configural process of geometrical reasoning of low and high groups are not different.

H012  There is no significant interaction between instructional strategies (DCM and CGL) with regard to geometry self-efficacy (high, intermediate and low) of students to yield different gain means on purely configural process of geometrical reasoning.

  H012.1  Through DCM, high, intermediate and low geometry self-efficacy students do not differ in their purely configural process of geometrical reasoning gain means.

  H012.2  Through CGL, high, intermediate and low geometry self-efficacy students do not differ in their purely configural process of geometrical reasoning gain means.

  H012.3  High geometry self-efficacy students achieve equal purely configural process of geometrical reasoning gain means in DCM and CGL.
**H012.4** Intermediate geometry self-efficacy students achieve equal purely configural process of geometrical reasoning gain means in DCM and CGL.

**H012.5** Low geometry self-efficacy students achieve equal purely configural process of geometrical reasoning gain means in DCM and CGL.

**Natural Discursive Process of Geometrical Reasoning**

**H013** There is no significant difference in gain mean of students on natural discursive process of geometrical reasoning studying through DCM and CGL.

**H014** There is no significant difference in gain mean of students on natural discursive process of geometrical reasoning of students with high, intermediate and low geometry self-efficacy groups.

**H014.1** Gain means of students on natural discursive process of geometrical reasoning of high and intermediate groups are not different.

**H014.2** Gain means of students on natural discursive process of geometrical reasoning of intermediate and low groups are not different.

**H014.3** Gain means of students on natural discursive process of geometrical reasoning of low and high groups are not different.

**H015** There is no significant interaction between of instructional strategies (DCM and CGL) with geometry self-efficacy (high, intermediate and low) of students to yield different gain means on natural discursive process of geometrical reasoning.

**H015.1** Through DCM, high, intermediate and low geometry self-efficacy students do not differ in their natural discursive process of geometrical reasoning gain means.

**H015.2** Through CGL, high, intermediate and low self-efficacy students do not differ in their natural discursive process of geometrical reasoning gain means.
H015.3 High self-efficacy students achieve equal natural discursive process of geometrical reasoning gain means in DCM and CGL.

H015.4 Intermediate self-efficacy students achieve equal natural discursive process of geometrical reasoning gain means in DCM and CGL.

H015.5 Low self-efficacy students achieve equal natural discursive process of geometrical reasoning gain means in DCM and CGL.

Theoretical Discursive Process of Geometrical Reasoning

H016 There is no significant difference in total gain mean of students on theoretical discursive process of geometrical reasoning studying through DCM and CGL.

H017 There is no significant difference in total gain mean of students on theoretical discursive process geometrical reasoning of students with high, intermediate and low geometry self-efficacy groups.

H017.1 Gain means of students on theoretical discursive process geometrical reasoning of high and intermediate groups are not different.

H017.2 Gain means of students on theoretical discursive process geometrical reasoning of intermediate and low groups are not different.

H017.3 Gain means of students on theoretical discursive process geometrical reasoning of low and high groups are not different.

H018 There is no significant interaction effect between instructional strategies (DCM and CGL) with regard to geometry self-efficacy (high, intermediate and low) of students to yield different total gain means on theoretical discursive process geometrical reasoning.

H018.1 Through DCM, high, intermediate and low geometry self-efficacy students do not differ in their theoretical discursive process geometrical reasoning gain means.

H018.2 Through CGL, high, intermediate and low geometry self-efficacy students do not differ in their theoretical discursive process of geometrical reasoning gain means.
**H018.3** High geometry self-efficacy students achieve equal theoretical discursive process geometrical reasoning gain means in DCM and CGL.

**H018.4** Intermediate geometry self-efficacy students achieve equal theoretical discursive process geometrical reasoning gain means in DCM and CGL.

**H018.5** Low geometry self-efficacy students achieve equal theoretical discursive process geometrical reasoning gain means in DCM and CGL.

**Criterion Performance (Total)**

**H019** There is no significant difference in total gain mean of students on criterion performance (total) studying through Duval’s Cognitive Model and conventional group learning.

**H020** There is no significant difference in total gain mean of students on criterion performance (total) of students with high, intermediate and low geometry self-efficacy groups.

**H020.1** Gain means of students on criterion performance of high and intermediate groups are not different.

**H020.2** Gain means of students on criterion performance of intermediate and low groups are not different.

**H020.3** Gain means of students on criterion performance of low and high groups are not different.

**H021** There is no significant interaction effect of instructional strategies (Duval’s Cognitive Model and conventional group learning) with geometry self-efficacy (HGSE, IGSE and LGSE) of students to yield different total gain means on criterion performance.

**H021.1** Through Duval’s Cognitive Model: high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance gain means.
H021.2 Through conventional group learning: high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance gain means.

H021.3 High Geometry self-efficacy students achieve equal criterion performance gain means in Duval’s Cognitive Model and conventional group learning.


H021.5 Low Geometry self-efficacy students achieve equal criterion performance gain means in Duval’s Cognitive Model and conventional group learning.

Criterion Performance (Objective)

H022 There is no significant difference in total gain means of students on objective type criterion performance or criterion performance (objective) studying through Duval’s Cognitive Model and conventional group learning.

H023 There is no significant difference in total gain mean of students on objective type criterion performance of students with high, intermediate and low geometry self-efficacy groups.

H023.1 Gain means of students on criterion performance (objective) of high and intermediate groups are not different.

H023.2 Gain means of students on objective type criterion performance of intermediate and low groups are not different.

H023.3 Gain means of students on objective type criterion performance of low and high groups are not different.

H024 There is no significant interaction effect of instructional strategies (Duval’s Cognitive Model and conventional group learning) with geometry self-efficacy (HSE, ISE and LSE) of students to yield different total gain means on criterion performance (objective).
H024.1 Through Duval’s Cognitive Model, high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance (objective) gain means.

H024.2 Through conventional group learning, high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance (objective) gain means.

H024.3 High Geometry self-efficacy students achieve equal criterion performance (objective) gain means in Duval’s Cognitive Model and conventional group learning.

H024.4 Intermediate Geometry self-efficacy students achieve equal criterion performance (objective) gain means in Duval’s Cognitive Model and conventional group learning.

H024.5 Low Geometry self-efficacy students achieve equal criterion performance (objective) gain means in Duval’s Cognitive Model and conventional group learning.

**Criterion Performance (Subjective)**

H025 There is no significant difference in total gain mean of students on criterion performance (subjective) studying through Duval’s Cognitive Model and conventional group learning.

H026 There is no significant difference in total gain mean of students on criterion performance (subjective) of students with high, intermediate and low geometry self-efficacy groups.

H026.1 Gain means of students on criterion performance (subjective) of high and intermediate groups are not different.

H026.2 Gain means of students on criterion performance (subjective) of intermediate and low groups are not different.

H026.3 Gain means of students on criterion performance (subjective) of low and high groups are not different.

H027 There is no significant interaction effect of instructional strategies (Duval’s Cognitive Model and conventional group learning) with geometry self-
efficacy (HSE, ISE and LSE) of students to yield different total gain means on criterion performance (subjective).

**H027.1** Through Duval’s Cognitive Model: high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance (subjective) gain means.

**H027.2** Through conventional group learning: high, intermediate and low Geometry self-efficacy students do not differ in their criterion performance (subjective) gain means.

**H027.3** High Geometry self-efficacy students achieve equal criterion performance (subjective) gain means in Duval’s Cognitive Model and conventional group learning.

**H027.4** Intermediate Geometry self-efficacy students achieve equal criterion performance (subjective) gain means in Duval’s Cognitive Model and conventional group learning.

**H027.5** Low Geometry self-efficacy students achieve equal criterion performance (subjective) gain means in Duval’s Cognitive Model and conventional group learning.

6.11 METHODOLOGY

6.11.1 Tools used

The following tools were used for collecting data:

- **Entry Geometry Test**: Developed and validated by the investigator.
- **Duval Cognitive Model Instructional Packages**: Developed and validated by the investigator.
- **Formative tests**: Developed and validated by the investigator.
- **Criterion test for summative evaluation**: objective, subjective type Summative test developed and validated by the investigator.
- **Geometrical Reasoning Test**: Developed and validated by the investigator.
- **Geometry Self-Efficacy Scale**: Developed and validated by the investigator.

6.11.2 Sample

The sample in the present was drawn at two levels:
• School sample
• Student sample

**The school sample**

The school sample was drawn from the representative secondary schools of Union Territory of Chandigarh where the medium of instruction was Hindi and schools were of co-educational type. The list of the schools under the administration of the Union Territory of Chandigarh was procured from the Director Public Instructions (schools) through the District Education Officer. The schools were compared with regard to following criteria: schools had almost the same classroom climate, physical facilities; teacher-taught ratio etc. It was ensured that all the schools were of secondary school level, co-educational and Hindi medium affiliated to Central Board of Secondary Education (CBSE). A list of such schools was formed and three schools were randomly selected from this list. One of these schools was again randomly allocated to experimental and control group. Thus three schools, which fulfil the criteria, were approached for seeking the permission to conduct the experiment.

The following three schools were drawn for the data collection:

• Govt. Senior Secondary School, Sector 15, Chandigarh
• Govt. High School, Sector 24, Chandigarh
• Govt. High school, Sector 25, Chandigarh

**Student sample**

Out of randomly selected three schools, initial sample of 319 students of class VIII of mathematics were selected randomly. Each of the selected three schools had more than one sections of eighth grade students. Hence two sections from Govt. Senior secondary school, sector 15, two section from Govt. High school, sector 24 and Govt. High school, sector 25 (four sections) with N=158 were randomly considered as control group and with N=161 were selected as experimental group for conducting the experiment.

Distribution of the Final Sample (N=245) according to Geometry Self-Efficacy has been given below:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Treatment</th>
<th>High</th>
<th>Intermediate</th>
<th>Low</th>
<th>Total</th>
</tr>
</thead>
</table>

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### Design of the Study

The present study employed an experimental method with pre-test post-test control group design. It employed a 2x3 factorial design wherein the data were analysed through 2x3 ANOVA. The variables under study are discussed below:

- **Independent variable**: instructional strategy was treatment variable which was studied at two levels:
  - *Experimental group*—was taught through Duval’s cognitive Model (DCM)
  - *Control group*—was taught through conventional group learning (CGL)

- **Dependent variable**: effect of independent variables was studied on dependent variables i.e. geometrical reasoning and achievement.

- **Classifying variable**: the students were classified on the basis of their geometry self-efficacy. It was studied at three levels: high geometry self-efficacy, intermediate geometry self-efficacy and low geometry self-efficacy.

The schematic layout of design has been given in figure 6.1:
6.11.4 Procedure

Procedure of the experiment comprised of two main stages, which were:

**Stage I**: Selecting the sample

**Stage II**: Conducting the Experiment

**Stage I: Selecting the Sample**

The sample was selected at two levels: the school level and the students’ level. Three schools with 319 students were selected for conducting the experiment. The procedures adopted for the selection of sample have already been discussed under the heading sample.

**Stage II: Conducting the Experiment**

The experiment was conducted in five phases as stated below:

**Phase I**: Administration of the pre-requisite skill tests

- Pre-requisite test (objective, subjective)

**Phase II**: Administration of the pre-test and other classification instruments

- Criterion Test
- Geometrical Reasoning Test and
- Geometry Self-Efficacy Scale

**Phase III**: Implementing the instructional programme

- Implementing the Duval’s cognitive model (DCM).
Phase IV: Administration of the post-test

- A criterion post-test and
- Post-test of Geometrical Reasoning

Phase V: Scoring, Tabulation and Analyses of the data

The detail of all the phases has been discussed below:

Phase I: Administration of the Entry Geometry Test

Before implementing the instructional programme to the experimental group, all the students were Pre-requisite Tests (Entry Geometry Test) to establish the equivalence of level of entry behaviour status of the students in the two instructional groups.

Phase II: Administration of the pre-test and other classification instruments

After administering EGT all the students of both the groups were administered the criterion test (summative test), Geometrical Reasoning Test as pre-test and Geometry self-efficacy scale for classifying the groups.

Phase III: Implementing the instructional programme

For the experiment, the students were classified on the basis of self-efficacy and then were divided into two groups; the experimental and control group. The experimental group learnt through DCM while the control group was taught through conventional strategy.

For the Experimental Group

The instruction treatment was administered according to the following steps:

Step I: Focus

The students were informed of the objectives of each lesson. In this step, the investigator ensured that the students would learn best when they receive proper attention and care from her.

Step II: Syntax

Syntax of DCM

Phase I: Visualization Processes

Visualization process refers to space representation for illustration, exploration or verification of different geometric situation. It include following activities:
(Activities)  
(i) Perceptual apprehension  
(ii) Discursive apprehension  
(iii) Operative apprehension  

Phase II: Construction Processes  
Construction processes are related to action for constructing a configuration according to restricted tools and geometrical requirements. This phase includes the following activities:  

(Activities)  
(i) Construction of concept via mind (mental imagery)  
(ii) Construction using tools: ruler and compass etc.  

Phase III: Reasoning Processes  
Reasoning process is related to discursive processes for proof and explanation. This phase includes activities as under:  

(Activities)  
(i) Purely configural process  
(ii) Natural discursive process  
(iii) Theoretical discursive process  

Materials used for instruction  
- The students were initiated to interact through visual aids like charts, models, moving models (perceptual apprehension) along either with teacher explanation, supporting/contradicting the statements (discursive apprehension); or manipulating the figure/diagrams (operative apprehension).  
- Chalkboard, coloured chalks, geometry instruments were utilised for drawing/constructing figures and for solving the questions.  
- The investigator creates situations, ask learners to classify or ask solution for questions which enforces the children to think critically so that reasoning ability was developed.  

Step III: Formative test  
A unit formative test was administered [appendices A-4(i to xvii)] to all the students after 2 or 3 lessons.  

Step IV: Repeat the cycle  
After ensuring learning on unit I, the students were presented with unit II. The cycle of focus, syntax, formative test as a feedback was repeated and the investigator completed all the five instructional units.
Step V: Summative Test

After having complete content, a summative test was administered to this group. The total time of implementing treatment was around two months.

Principles of reaction

This element is concerned with the reaction of the teacher to the response of the students. In DCM, the teacher’s task is to nurture the reasoning ability in the students by emphasizing the cognitive interaction processes. The teacher need to appreciate student’s ideas, argument, and concepts about geometrical figures/ shapes in terms of mathematical language and symbols. During phase I and phase II, teacher is to remain supportive for turning the student’s attention towards analysis of visual imagery, visual perception and construction of geometrical figures. During phase III, teacher task is to encourage and help student to develop different ways thinking/ reasoning.

Social system

It refers to the relationship between students and teacher. It describes the role of the students and the teacher and interrelationships within the classroom and outside it. In DCM, social system is comprised of mixed interaction between teacher and students. In phase I and phase II of syntax i.e. visualisation and construction processes both teacher and students is equally active but in the phase III i.e. reasoning processes students are more active than teacher. Here, the question of passive learner is ruled out. The students can express themselves freely and can make conclusions. Ample freedom is given to the students for carrying out their own thinking. They may formulate their own ideas, concepts for making argument and test these in the light of the available data. It is however true that teacher formulates instructional objectives, decides about the instructional process to be carried out in the limelight of this model. In the whole, teacher acts as a facilitator with democratic attitude in order to attain equal sharing between her and the students.

Support system

This component is used to describe the supporting conditions necessary to attain the model goal. It describes that what additional equipment in addition to the school will throw more light on the subject. Apart from ordinary nicely equipped classroom with blackboard, chalk, pointer etc. in DCM, teacher requires resource material like moving model, charts, geometry instruments, paper cut-outs, daily life
used material cube, cuboids and cylindrical shaped box and puzzle toy etc. to provide maximum support to the learners to encounter to the problem and to develop reasoning ability through visual imagery, mental imagery and drawing geometrical figures/shapes. In order to make the concepts clearer to the learner, to check their performance/achievement of the objective and feedback home assignment, formative test will be given to the learners.

**Instructional and Nurturant effects**

Each model is developed around some focus/goal. The success of the model is measured by the extent it has attained the goal. But every model affects some other aspects of the student’s behaviour also. The primary purpose of DCM is to teach learners how to reflect on the reasoning and inquiry, how to define their problems, how to work with others in exploring facts, different ways of looking at geometrical figures and how to conclude on the basis of available. Besides achievement, instructions given in the instructional plan is used to develop reasoning ability, visualisation and construction skills. Here teacher plans some activities and create a climate that nurture understanding of facts and ideas, visual imagery, logical reasoning, openness as well as cognitive interaction processes. It also nurtures a spirit of cooperation and as ability to work with other in a reasoning process.

**For control group**

The instructional package based on conventional group learning was administered according to the following plan:

**Step I:** The objective and content outlines of forty lessons were provided to the mathematics (control group) teacher by the investigator, so that there is no difference among groups on the amount of content taught to them.

**Step II:** This group was taught by their mathematics teacher in the conventional group learning situation. It generally refers to chalk and talk method.

**Step III:** No formative tests were conducted after the completion of different units. The time schedule followed for this group was similar to that of the other two groups.

**Step IV:** After having complete content, a summative test was administered to this group also.
Phase IV: Administration of the Post-test

After completion of all the instructional units, a summative test (objective, subjective) and geometrical reasoning test were administered to all the students. At the end, students were thanked for their full cooperation.

Phase V: Scoring, Tabulation and Analyses of the data

The responses of all the students were scored according to prescribed scoring keys of pre-test, self-efficacy scale, and post-test. The scores thus obtained were subjected to statistical analysis.

6.11.5 Statistical Techniques

Following statistical techniques were employed to analyse the data to test the hypotheses:

1. Mean and standard deviations were used wherever required.
2. Graphical presentations, bar graphs, frequency polygon were drawn.
3. 2x3 analyses of variances on scores of achievement and geometrical reasoning test in relation to geometry self-efficacy were used.
4. Each significant F-ratios were followed by t-test whenever required.

6.12 MAJOR FINDINGS

In the light of the interpretation of the results of the present study, the following conclusions were drawn:

Section II (Part A)

Part A of section II consisted of analyses related to scores of Geometrical Reasoning. Major findings of which have been reported under the following subheading:

❖ Results Based on the Analysis of Gain Score on Geometrical Reasoning (total)

- Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on geometrical reasoning.
- High geometry self-efficacy of students did not yield significantly different geometrical reasoning gain means when compared with that of intermediate geometry self-efficacy.
- The students of high geometry self-efficacy yielded higher geometrical reasoning gain means than low geometry self-efficacy.
The students of Intermediate geometry self-efficacy yielded higher geometrical reasoning gain means than low geometry self-efficacy.

Through DCM, the difference in geometrical reasoning gain means for high geometry self-efficacy and intermediate geometry self-efficacy.

Through DCM, the difference in geometrical reasoning gain means for high geometry self-efficacy and low geometry self-efficacy were significantly different.

Through DCM, intermediate geometry self-efficacy scored higher gain mean than low geometry self-efficacy.

Through CGL, with high, intermediate and low geometry self-efficacy achieved equal geometrical reasoning gain means.

Students with High geometry self-efficacy achieved higher geometrical reasoning gain means through DCM than CGL.

Students With intermediate geometry self-efficacy achieved higher geometrical reasoning gain means through DCM than CGL.

Students With low geometry self-efficacy achieved higher geometrical reasoning gain means through DCM as compared to their counterparts in the CGL situation.

Results Based on the Analysis of Gain Score on Purely Configural Process of geometrical reasoning

Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on purely configural process of geometrical reasoning.

High geometry self-efficacy of students did not yield significantly different purely configural process of geometrical reasoning gain means when compared with that of intermediate geometry self-efficacy.

The students of high geometry self-efficacy yielded higher purely configural process gain means than low geometry self-efficacy.

Intermediate geometry self-efficacy of students did not yield significantly different purely configural process gain means when compared with that of low geometry self-efficacy.

For gain mean scores of purely configural process, the two variables under study were independent of each other.
Results Based on the Analysis of Gain Score on Natural Discursive Process of geometrical reasoning

- Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on geometrical reasoning.
- High geometry self-efficacy of students did not yield significantly different geometrical reasoning gain means when compared with that of intermediate geometry self-efficacy.
- The students of high geometry self-efficacy yielded higher geometrical reasoning gain means than low geometry self-efficacy.
- The students of Intermediate geometry self-efficacy yielded higher geometrical reasoning gain means than low geometry self-efficacy.
- Through DCM, the comparable geometrical reasoning gain means for high geometry self-efficacy and low geometry self-efficacy.
- Through DCM, the difference in geometrical reasoning gain means for high geometry self-efficacy and low geometry self-efficacy were significantly different.
- Through DCM, intermediate geometry self-efficacy scored higher gain mean than low geometry self-efficacy.
- Through CGL, with high, intermediate and low geometry self-efficacy achieved equal geometrical reasoning gain means.
- Students with high self-efficacy achieved higher geometrical reasoning gain means through DCM than CGL.
- Students With intermediate self-efficacy achieved higher geometrical reasoning gain means through DCM than CGL.
- Students With low self-efficacy achieved higher geometrical reasoning gain means through DCM as compared to their counterparts in the CGL situation.

Results Based on the Analysis of Gain Score on theoretical discursive process of geometrical reasoning

- Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on theoretical discursive process of geometrical reasoning.
High geometry self-efficacy of students did not yield significantly different theoretical discursive process gain means when compared with that of intermediate geometry self-efficacy.

The students of high geometry self-efficacy yielded higher theoretical discursive process of geometrical reasoning gain means than low geometry self-efficacy.

The students of Intermediate geometry self-efficacy yielded higher theoretical discursive process of geometrical reasoning gain means than low geometry self-efficacy.

Through DCM, the comparable theoretical discursive process of geometrical reasoning gain means for high geometry self-efficacy and intermediate geometry self-efficacy.

Through DCM, the difference in theoretical discursive process of geometrical reasoning gain means for high geometry self-efficacy and low geometry self-efficacy were significantly different.

Through DCM, intermediate geometry self-efficacy scored higher gain mean than low geometry self-efficacy.

Through CGL, with high, intermediate and low geometry self-efficacy achieved equal theoretical discursive process of geometrical reasoning gain means.

Students with high self-efficacy achieved higher theoretical discursive process of geometrical reasoning gain means through DCM than CGL.

Students with intermediate self-efficacy achieved higher theoretical discursive process of geometrical reasoning gain means through DCM than CGL.

Students with low self-efficacy achieved higher theoretical discursive process of geometrical reasoning gain means through DCM as compared to their counterparts in the CGL situation.

**Section II (Part B)**

Part B of section II consisted of analyses related to scores of Criterion Performance. Major findings of which have been reported under the following subheading:
Results Based on the Analysis of Gain Score on criterion performance

(Total)

- Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on criterion performance.
- High geometry self-efficacy of students did not yield significantly different criterion performance gain means when compared with that of intermediate geometry self-efficacy.
- The students of high geometry self-efficacy yielded higher criterion performance gain means than low geometry self-efficacy.
- The students of Intermediate geometry self-efficacy yielded higher criterion performance gain means than low geometry self-efficacy.
- Through DCM, the comparable criterion gain means for high geometry self-efficacy and intermediate geometry self-efficacy.
- Through DCM, the difference in criterion performance gain means for high geometry self-efficacy and low geometry self-efficacy were significantly different.
- Through DCM, intermediate geometry self-efficacy scored higher gain mean than low geometry self-efficacy.
- Through CGL, with high, intermediate and low geometry self-efficacy achieved equal criterion performance gain means.
- Students with high self-efficacy achieved higher criterion performance gain means through DCM than CGL.
- Students With intermediate self-efficacy achieved higher criterion performance gain means through DCM than CGL.
- Students With low self-efficacy achieved higher criterion performance gain means through DCM as compared to their counterparts in the CGL situation.

Results Based on the Analysis of Gain Score on criterion performance

(objective)

- Students studying through DCM achieved higher gain means than those who were studying in CGL situation, on criterion performance (objective).
High geometry self-efficacy of students did not yield significantly different criterion performance (objective) gain means when compared with that of intermediate geometry self-efficacy.

The students of high geometry self-efficacy yielded higher criterion performance (objective) gain means than low geometry self-efficacy.

The students of Intermediate geometry self-efficacy yielded higher criterion performance (objective) gain means than low geometry self-efficacy.

For gain scores of criterion performance (objective) the two variables under study were independent of each other.

Results Based on the Analysis of Gain Score on criterion performance (subjective)

Students studying through DCM achieved higher gain means than those who were studying in a CGL situation, on criterion performance (subjective).

High geometry self-efficacy of students did not yield significantly different criterion performance gain means when compared with that of intermediate geometry self-efficacy.

The students of high geometry self-efficacy yielded higher criterion performance (subjective) gain means than low geometry self-efficacy.

The students of Intermediate geometry self-efficacy yielded higher criterion performance (subjective) gain means than low geometry self-efficacy.

Through DCM, the comparable criterion gain means for high geometry self-efficacy and intermediate geometry self-efficacy.

Through DCM, the difference in criterion performance (subjective) gain means for high geometry self-efficacy and low geometry self-efficacy were significantly different.

Through DCM, intermediate geometry self-efficacy scored higher gain mean than low geometry self-efficacy.

Through CGL, with high, intermediate and low geometry self-efficacy achieved equal criterion performance (subjective) gain means.

Students with high self-efficacy achieved higher criterion performance (subjective) gain means through DCM than CGL.
Students With intermediate self-efficacy achieved higher criterion performance (subjective) gain means through DCM than CGL.

Students With low self-efficacy achieved higher criterion performance (subjective) gain means through DCM as compared to their counterparts in the CGL situation.

6.13 EDUCATIONAL IMPLICATIONS

The results of the present study supported that the Duval’s cognitive model may be used to enhance the geometrical reasoning of students in math as compared to the traditional method of teaching. It is evident from the research that if teacher try to switch over to Duval’s cognitive model of teaching, not only ability of reasoning will improve but will also improve learning outcomes of the students in mathematics/geometry.

The school administrators and teachers can use this cognitive teaching model in furtherance of mathematics education at secondary stage. Since Duval’s cognitive model was found to be the most effective model of teaching geometry, the mathematic teachers should be provided intensive training in the use of Duval’s cognitive model, and in the development of instructional material to be employed while using the Duval’s cognitive model. National Ministry of Education should provide in-service training for teachers to introduce this model of teaching to teach mathematics.

Most Mathematics classes in India are stress, anxiety and phobic prone especially for learning reasoning/proof in Euclidean school geometry. The cognitive model can be immense use if implemented in the present day Indian school system under new scheme of CCE which aim to enhance child’s cognitive, affective aspects. In the present study, Duval’s cognitive model was found more effective as compared to conventional group learning for development of geometrical reasoning in total scores, for scores of purely discursive process, for natural discursive process and for theoretical discursive process. This cognitive model of teaching mathematics/geometry tried to express the possibilities of dissipating stress, anxiety and phobia, especially academic stress, achievement stress and mathematics anxiety. It may be suggested that School administrators should help teachers in implementing this cognitive model based lesson plans. For this administrators could organise workshops about how to put into practice a cognitive model based lesson in math class.
It has been realized by educationists’ worldwide that quality of education be raised by incorporating models based teaching, which boost students’ achievement in academic setting. The difference in achievement gain means for total, for objective and for subjective of the two instructional strategies (DCM and CGL) suggested that instruction through Duval’s cognitive model result into better achievement as compared to conventional group learning. The result may be significant help for Curriculum developers to consider/view various activities involved in syntax of Duval’s cognitive model during curriculum development process. They could involve cognitive model based instruction as a teaching method in new curricula of mathematics.

The result can also be useful for school administration to have mixed groups of students having high, intermediate and low self-efficacy. Since Duval’s cognitive model has proved to enhances geometrical reasoning and achievement of all students whether they have high, intermediate and low geometry self-efficacy. The results of present investigation have encouraging solution for administrators in a way that students with varied self-belief of present day scenario of Indian education system put more emphasis on levels of achievement. It is also recommended that teachers participate in a training programme to become familiar with different cognitive models of teaching mathematics and their effect on student reasoning ability and achievement so that they may improve learning in any classroom.

Experiences of success or failure have important effects on a child's perceptions and beliefs of their abilities and their future expectations in many achievement settings. Teachers need to provide many opportunities for children to experience success. Children must be encouraged to perceive themselves as capable learners, to set high standards, believe in themselves, and to develop learning strategies to help them overcome difficulties. It is important that successful opportunities are provided for all students. Hence efforts should be made by teachers to enhance students’ Self-efficacy or beliefs in their own capability, to impel or propel engagement in the learning process, and to teach students through relevant strategies that can be used.

Thus, teacher dominated classroom environment with maximum focus on content, chalk and talk may be erased through teaching with cognitive model, Which would not only make classroom climate lively and participative but will help in
building self-confidence and enhancing reasoning ability. All these will prepare our students for meeting the challenges of life courageously, confidently and successfully.

6.14 SUGGESTIONS FOR FURTHER RESEARCH

The investigator is quite aware of the limitations under which the present investigation was conducted and therefore that no sweeping generalizations could be made. The findings are only indicative of trends and hence are to be viewed in light of following limitations:

• The sample of the student was limited to 245 students drawn mainly from the Hindi medium Govt. School having well developed infrastructures.
• The sample was limited only to the union territory of Chandigarh.
• The study was limited to only grade VIII students.
• The variables studied were limited to Duval’s cognitive model, geometrical reasoning, achievement and geometry self-efficacy.
• Study was conducted on both boys and girls.
• Out of three cognitive processes of Duval’s cognitive model only geometrical reasoning processes has been selected for the study.

The investigator, by virtue of her experience in the field of study humbly offers the following suggestions for further research that could be taken by the perspective researchers.

• Based on the present research about Duval’s cognitive model, geometrical reasoning, achievement and geometry self-efficacy, it is clear that experimental group (instruction through Duval’s cognitive model) seemed to be better in achieving higher gains on geometrical reasoning and geometry performance as compared to control group. Effectiveness of Duval’s cognitive model may be researched further at large scale for large number of students of different age groups, grade levels, ability level and socio-economic status.
• Based on these current findings, future studies could investigate the relationship between geometrical reasoning, achievement of students in geometry.
• Other models belonging to cognitive model of teaching can be explored by implementing them in the classroom situations and comparative studies may be taken up.
• Similar studies can be designed for other topics of mathematics such as algebra, statistics, trigonometry etc. and can be tried at different levels of education.

• Further studies may be planned and conducted around affective variables like self-concept, self-esteem, levels of aspiration, math anxiety, math self-efficacy, and other personality traits.

• For greater validity of results and for arriving at conclusive generalizations the study may be replicated on larger population.

• The present study was confined to Govt. Hindi medium students only. Further research can be carried out to compare the effect of Duval’s cognitive model on the students of Govt. and private schools, rural and urban areas.

• Research can be carried out to study the effect of different instructional strategies using theoretical framework of Duval’s cognitive model.

• The effectiveness of technology based instruction such as dynamic geometry software; 3Dcabari etc. using theoretical framework of Duval’s cognitive model can be studies at secondary or higher level.

• Effectiveness of Duval’s cognitive model may be researched further at larger scale for large number of learner of different age groups and grade levels.

• Further investigation is also suggested for other cognitive processes of Duval’s cognitive model such as visualization processes and construction processes.

• Further research can be carried out to investigate the effect of Duval’s cognitive model on geometry self-efficacy of students.

• Effectiveness of Duval’s cognitive model may also be verified for different population such as slow learners, special students, students with learning disabilities (dyscalculia) and tactile type learners.

• Meta-analysis of the studies in respect of Duval’s cognitive model may be conducted.

This is not an exhaustive list. However, a few of them have been enumerated above in order to indicate the possible studies that could be undertaken immediately in this important area of cognitive model for integrated development of children. Thus research studies in this area evince good scope and will continue to make notable contributions in the future.